

HERTZIAN POINT CONTACT - SLIDING AND ROLLING ASPECTS

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Abstract: *The paper presents a dynamical simulation of the dry contact between a ball and outer race, in vertical plane, with the purpose of pure rolling identification. The software employed in simulation doesn't require explicitly the coefficient of rolling friction, but, instead, a damping coefficient is used. The results of simulation were compared to the experimental results obtained on a test rig. The comparison is helpful because allows correlation between the experimental rolling friction coefficient and damping coefficient used in simulation.*

Key words: *rolling friction, simulation, test rig*

1. Introduction

Sliding, rolling or rolling with sliding motion is met in most of the machines and mechanisms from technical applications or everyday life. Rolling friction and sliding friction as complex physical-chemical processes detrimentally influence motion and efficiency of mechanical transmissions. A specific situation concerning the study of rolling motion is the beginning phase of rolling motion, when the rolling torque is smaller than the total friction torque and the body presents only the tendency of rolling.

A model used in the study of rolling motion is represented by the assembly between ball and ring bearing, positioned in a vertical plane, when the ball is set free on the outer race, [1], [2].

2. Theoretical background

When the ring is rotating with controlled angular velocity, leads to the occurrence of a friction force that alters the balance of the body, figure 1. The friction from contact acts as traction upon the ball which is perturbed from the static equilibrium state at zero speed and thus the centre of the ball will take a new position situated on a direction making an angle with the vertical. Under these circumstances, a couple occurs and the ball tends to roll, similarly to the situation when the ball is positioned on an inclined plane. Due to rolling tendency of the ball, the pressure distribution over contact area is asymmetrical and therefore the normal reaction, N is eccentric with respect to the centres line, with a quantity denoted s , [3], [4]. To describe the motion of the ball, the hypothesis of rigid body is assumed and one can apply the following theorems, [5]:

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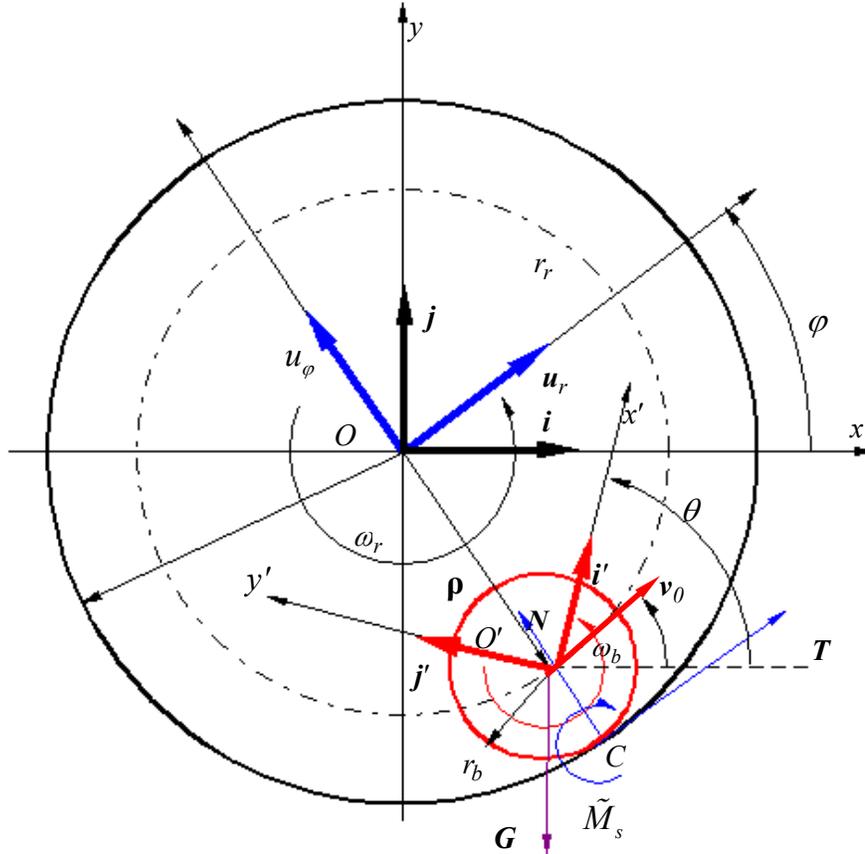


Fig. 1. Forces acting in ball-ring contact for the case of rotating ring

- The centre of mass theorem:

$$m\ddot{\rho} = \mathbf{G} + \mathbf{N} + \mathbf{T} \quad (1)$$

where:

$$\rho = \overline{OO'} \quad (2)$$

Is it convenient to write the projections of the equation (1) on the axis of mobile reference system, Fig. 1, having the unit vectors \mathbf{u}_r , (radial direction) and \mathbf{u}_φ (tangential direction):

$$\begin{cases} mr_r \ddot{\varphi} = T - mg \sin \varphi \\ 0 = N - mg \cos \varphi \end{cases} \quad (3)$$

- The moment of momentum theorem with respect to the centre of mass:

$$J_G \ddot{\theta} = T r_b - M_s \quad (4)$$

The motion of the ball cannot be completely described using relations (3) and (4). Two possible methodologies might be employed:

- The ball's motion on the race is a pure rolling one, then the relative velocity

between contacting points is expressed by:

$$v_{C_2 C_1} = 0 \quad (5)$$

- The ball's motion on the race is a sliding one, so it exists:

$$v_{C_2 C_1} \neq 0 \quad (6)$$

where C_1 and C_2 are the project arms of the point C on the bodies.

In the first situation, the unknowns of the system are: one of the angles of position corresponding to ball/race (the pure rolling condition imposes a relationship between the two parameters), the magnitude of rolling torque M_s that in this case is expressed as:

$$M_s = sN \quad (7)$$

and the size of friction force, T , that should satisfy the following condition:

$$T < \mu N \quad (8)$$

where μ is the boundary static friction coefficient.

In the second situation, when sliding is considered, the ball has two independent degrees of freedom (DOF) φ and θ , as shown in figure 1. The rolling torque is found using rel. (7) where the friction force results, in this case, from the relation:

$$T = \mu N \quad (9)$$

The decisive parameter selecting between rolling or sliding motion is the ratio T/N . Explicitly:

$$\begin{cases} T/N < \mu, & \text{pure rolling} \\ T/N = \mu, & \text{sliding} \end{cases} \quad (10)$$

3. Experimental Procedure

The test rig proposed by Musca, [6], [7], and improved by Siretean and Musca, [8], [9], presented in figure 2, is considered.

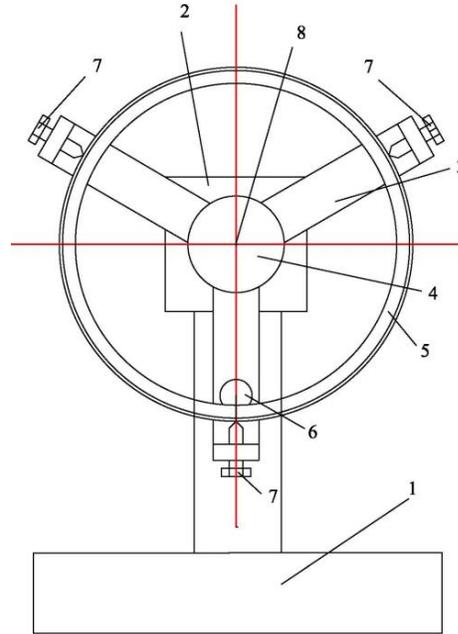


Fig. 2. *Experimental set-up for rolling friction estimation in a ball-outer race contact*

The experimental test rig is made of a ground structure, 1, on top of which a variable speed electrical motor, 2, with horizontal shaft is mounted. A gripper device, 3, rigidly mounted on the shaft, holds the outer bearing ring, 5. A free ball, 6, is placed on the outer race. The position of the ball can be identified by an equatorial circle traced on the ball's surface in proper position, which is analyzed from video-captures to find both the instant when rolling starts and the angular velocity during motion. A disc, 4, is fixed on the shaft, in the ring's symmetry plane, and radial and circular marks are traced for video analysis, used in finding the ring's centre of rotation and

instantaneous angular velocity. The coaxially between ring and motor shaft is obtained via screws 7. An optical device with laser rays is used to materialize the axis of fix system that projects on the disc 4, two perpendicular straight light beams, a horizontal and a vertical one. These systems allow identifying the position of centre of the ball and the ring. The orientation of the ball with respect to the system is found by the equatorial mark.

A high speed camera is used to acquire the ball's motion.

Specialised software is employed in image processing in order to find the frame corresponding to the initiation of ball's rolling. The image is exported to Autodesk Inventor, [8]. In this software, the position of the centre of a circle can be easily determined by tracing over the captured image three tangents to the circle. In this manner, the centres of the ring and ball are precisely found, figure 3.

The image of laser beams from the frame makes possible establishing the coordinate axis of the system. Tracing the straight line between the centres of the ball and the race and measuring the angle between it and the vertical, the position angle of the ball, α , is

precised. Once the ring starts to rotate, the friction force from contact operates upon the ball as active force, and the consequence is the displacement of contact point from static case.

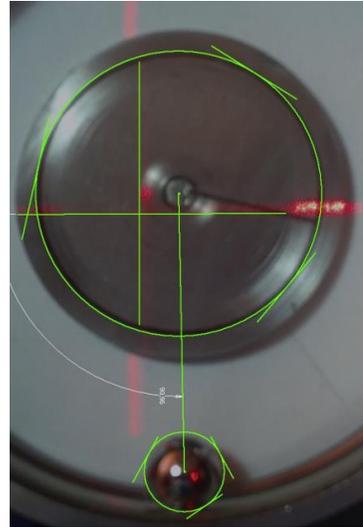


Fig. 3. *Ball's angle of deviation identification for dry conditions (lack of lubricant)*

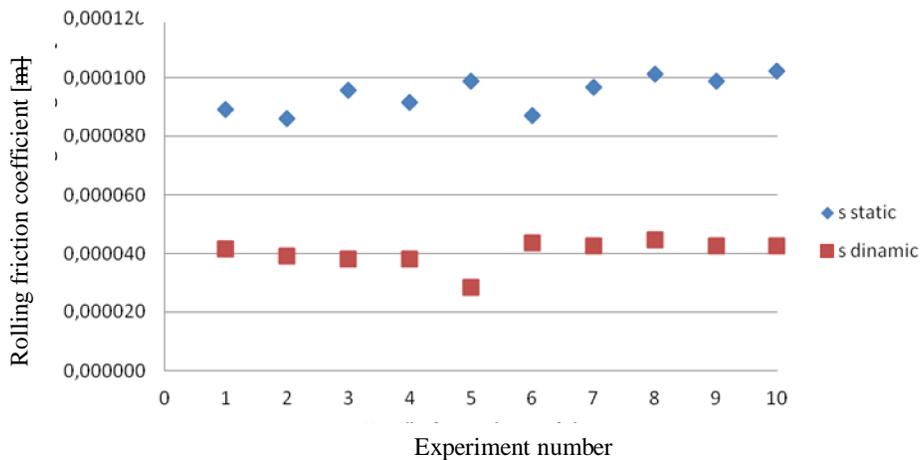


Fig. 4. *Static friction coefficient (rolling tendency) and dynamic friction coefficient (stable rotation velocity) for dry conditions - preliminary results*

To the tendency of motion of the centre of the ball the tangential component of ball's weight is opposing. In addition to the displacement of the ball's centre, rolling motion of the ball initiates, responsible being the torque of the friction force and the rolling friction torque. The rolling friction torque is proportional to normal load from contact, the proportionality factor being the rolling friction coefficient, s , having dimension of length. The rotation balance condition for the ball imposes the equality of the torques and as immediate consequence is the following relation:

$$s = r \cdot \operatorname{tg} \alpha \quad (11)$$

The tests were performed using a bearing outer ring with 110 mm diameter and a ball of 12 mm in diameter, under dry friction conditions, [9]. The results are presented in figure 4, [9], for two different situations: the instant of rolling motion start, denoted static rolling friction coefficient and for the case of steady rolling motion, respectively.

4. Dynamic system modelling

The dynamic behaviour of the system was simulated using MSC.ADAMS software. The two parts, the ring and the ball, were modelled, as presented in figure 5. The ring was jointed to the ground with a rotating joint, with imposed driving motion. The dry concentrated contact occurs between the ball and the outer race. In figure 6 there are presented the parameters required by software for tribological characterisation of the contact. One can notice that the software does not require explicit specification of coefficient of rolling friction, [10]. Thus, the rolling friction is characterised by damping coefficient and penetration depth. In order to obtain results comparable qualitatively to the experimental ones, it was simulated the ball motion for three different values of

damping coefficient, specifically 1, 10 and 100 to which, in the next plots, correspond the numbers 1,2 and 3 respectively. The other tribological parameters were maintained as in figure 6.

The first problem to be highlighted refers to the instant of ring's motion start. To avoid an initial shock, for the initial moment, was imposed that the angular velocity of the ring ought to be a quadratic function.

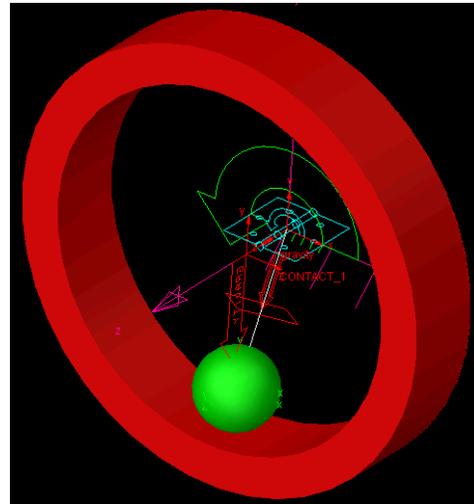


Fig. 5. *Ball-race system model*

Stiffness	1.0E+005
Force Exponent	2.2
Damping	100.0
Penetration Depth	0.1
<input type="checkbox"/> Augmented Lagrangian	
Friction Force	Coulomb
Coulomb Friction	On
Static Coefficient	0.3
Dynamic Coefficient	0.15
Stiction Transition Vel.	1.0
Friction Transition Vel.	1.0

Fig. 6. *Tribological parameters for ball-race contact*

A first set of envisaged parameters were the angular velocity of the ball and the velocity of the ball's centre. In Figures 7 and 8 there are presented the variations of these two parameters for the three damping coefficients mentioned. For a better visibility, the plots for the velocity of centre of mass were slide vertically.

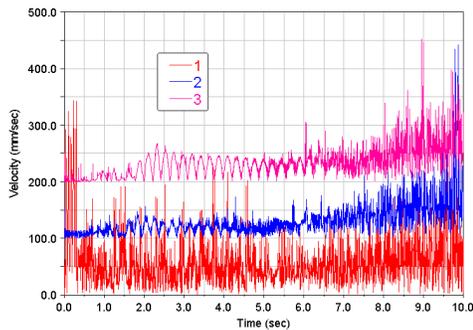


Fig. 7. *Velocity of ball's centre*

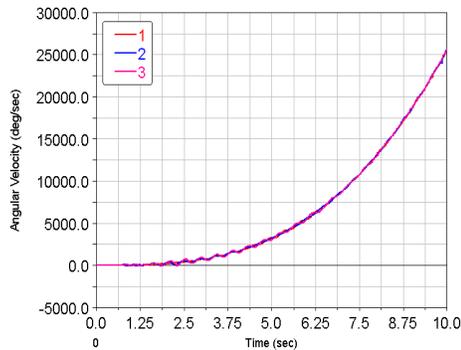


Fig. 8. *Angular velocity of the ball*

From figure 7 it can be observed that the velocity of the centre of mass varies around zero value, confirming the oscillatory motion of the ball centre. The noise from the plots may have as possible source the polyhedral approximation of the spherical surface.

One cannot draw a conclusion concerning rolling or sliding presence from the plots of velocity of ball's centre. While the velocity of ball's centre oscillates

around zero, the angular velocity of the ball increases monotonically regardless the damping coefficient value. The shape of these curves suggests a polynomial variation. Under the assumption of pure rolling motion, (condition 5), the ball and the ring performs similarly to a planetary geared mechanism. For such a mechanism, the transmission ratio must be constant, depending only on the geometry of the mechanism. To verify this hypothesis, in figure 9 it was represented the angular velocity of the ball versus the angular velocity of the ring. The proportionality between the two velocities is proved by the quasilinear aspect of this dependency. The deviation from straight line of the plots confirms the occurrence of sliding motion. Sliding is better highlighted in figure 10, where the variation of velocity of ball's centre versus the angular velocity of the ball was represented (vertically slide).

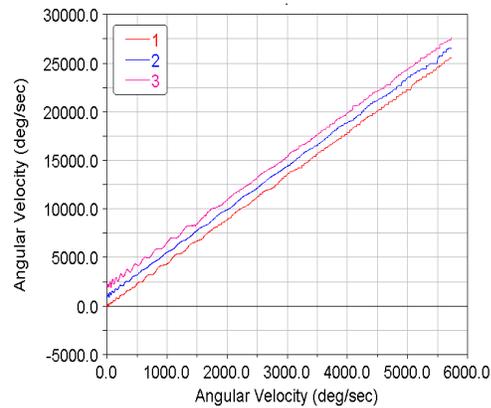


Fig. 9. *Angular velocity of the ball versus angular velocity of the ring*

Assuming pure rolling, according to Equation 5, these graphs should be represented by the same straight line, passing through origin.

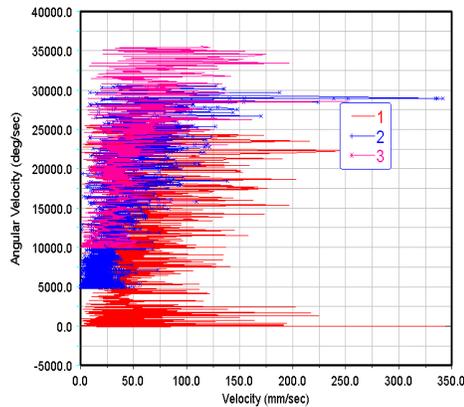


Fig. 10. Ball's angular velocity versus velocity of centre of mass

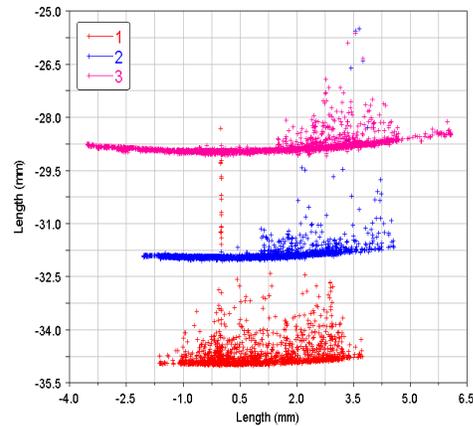


Fig. 11. Trajectories of ball's centre (slided)

5. Comparison between experimental and simulation results

As presented above, for modelling a rolling contact, the software doesn't require explicitly a rolling coefficient but a damping coefficient. Figure 11 presents the trajectories (vertically slide) of ball's centre and figure 12 shows the image of actual test rig, where the ball is in a random position after the ring reached the stationary velocity. It can be observed that this position is eccentric with respect to the vertical passing through the centre of the ring. This remark can be helpful when comparing the image with the trajectories of ball's centre presented in figure 11, (vertically slide).

One can notice that in all graphical representations, there are regions where the deviation from circular trajectory is maxim. In these regions the probability of losing contact is maxim. But, the probability of losing contact is maxim where the force ball-ring is minim. Since the normal force results from the ball weight and centrifugal force, one can conclude that the contact collapses in the zone where the velocity of the ball is minim (dynamic equilibrium position).

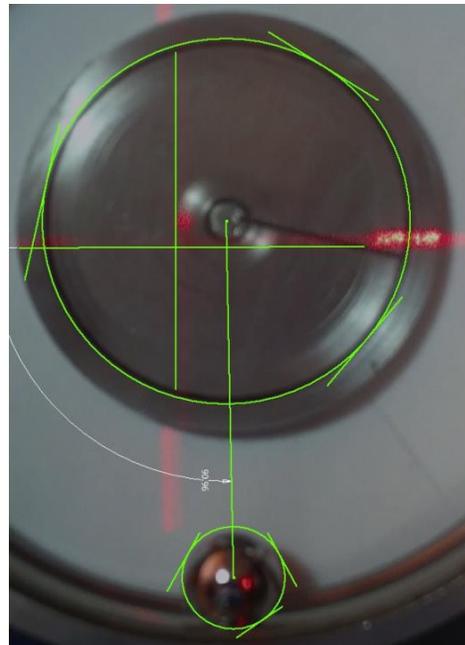


Fig. 12. Image of actual test rig- ball in a random position

Establishing the distance between the position of dynamic equilibrium and the vertical axis of the ring, it may be obtained a correlation between the rolling friction coefficient and damping coefficient from dynamical simulations.

6. Conclusions

The paper presents a dynamical simulation of the dry contact between a ball and outer race, in vertical plane, with the purpose of pure rolling identification.

The software employed in simulation doesn't require explicitly the coefficient of rolling friction, but, instead, a damping coefficient is used. The results of simulation were compared to the experimental results obtained on a test rig. The comparison is helpful because allows correlation between the experimental rolling friction coefficient and damping coefficient used in simulation.

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