

# COMPUTER PROGRAM FOR SINGLE SAMPLING PLANS BY VARIABLES WITH UNKNOWN LOT STANDARD DEVIATION

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**Abstract:** *The paper aims to conceive a computer program that can be used at the design as well as at the application of the single sampling plans by variables in the case when the lot standard is unknown. The developed program calculates first the plan parameters and then, by means of the sample measurements provided by the user, makes the decision about the lot acceptance or rejection. The program can be used for quality characteristics with one single or two specification limits. After the description of the program, examples that cover all cases of its use there are presented.*

**Key words:** *acceptance sampling, single sampling plans by variables, unknown standard deviation.*

## 1. Introduction

Acceptance sampling represents an important part of statistical quality control and some types of acceptance sampling plans have been studied in the last years literature.

For example, Wang (2018) proposed a type of sampling plan by variables for inflated-Pareto data that can reduce the sample size by 25-70% compared to the single sampling plan [8].

Klufa (2014) proposed a computer program that can be used for finding the optimal sampling plan for Dodge-Romig AOQL sampling plans for inspection by variables [2].

Santos-Fernandez, Govindaraju and Jones presented the design of sampling plans using composite samples and provided an implementation guide based on two points of the OC curve for perfect and imperfect mixing [4].

Lee, Wu and Wang (2018) proposed a modified sampling scheme by variables inspection based on the one-sided capability index [3].

Unlike bibliography above mentioned, the present paper considers another type of acceptance sampling plan, namely: the single sampling plan by variables in the case when the lot standard deviation  $\sigma$  is unknown, a plan that was not studied so much in the last years.

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There will be considered the cases when the quality characteristic may have one single specification limit (an upper limit  $U$  or a lower limit  $L$ ), or two specification limits.

One method that can be used for design and application of the single sampling plan by variables, is the  $k$  method that can be applied as follows [5].

Let us consider first the case when the quality characteristic has one single lower specification limit  $L$ . In order to perform the inspection, a random sample of size  $n$  is selected from the lot and the quality characteristic is measured. Then, using the measurements, the statistic  $z_L$  is calculated using the relation (1) [5]:

$$z_L = \frac{\bar{x} - L}{s}. \quad (1)$$

In relation (1),  $\bar{x}$  is the sample mean and  $s$  is the sample standard deviation.

If  $z_L$  is greater than or equal to the critical distance  $k$  the lot is accepted, otherwise the lot is rejected.

Analogous, if the quality characteristic has one single upper specification limit  $U$ , the  $z_U$  statistic is calculated using the formula (2) [5]:

$$z_U = \frac{U - \bar{x}}{s}. \quad (2)$$

If  $z_U$  is greater than or equal to the critical distance  $k$  the lot is accepted, otherwise the lot is rejected.

In the case when the quality characteristic has a lower specification limit  $L$  and an upper specification limit  $U$ , the application of the sampling plan becomes more difficult.

In this situation, the first step of plan application is the comparison of the sample standard deviation  $s$  with a maximum value allowed, which is denoted by  $MSD$  (maximum standard deviation).

In the case when  $s > MSD$  the lot is rejected because the value of the standard deviation is too high.

When  $s \leq MSD$ , a separate plan will be applied for each specification limit. If  $z_U \geq k$  and  $z_L \geq k$  the lot is accepted, otherwise the lot is rejected.

In order to calculate the value of maximum standard deviation  $MSD$ , the following steps must be fulfilled [7]:

- First it is calculated the value of  $p^{**}$ , which is the area under the normal curve placed on the right of  $z_p^* = k$ ;

- It is calculated the value of  $z_p^{**}$  so that the area under the normal curve placed on its right equals to  $p^{**}/2$ ;

- The value of the maximum standard deviation will be calculated using the formula (3) [7]:

$$MSD \cong \frac{U - L}{2z_p^{**}}. \quad (3)$$

When the standard deviation  $\sigma$  is unknown, the plan parameters which are the critical distance  $k$  and the sample size  $n$  are calculated by means of relations (4) and (5) respectively [6]:

$$k = \frac{z_{\alpha}z_2 + z_{\beta}z_1}{z_{\alpha} + z_{\beta}}, \quad (4)$$

$$n = \left(1 + \frac{k^2}{2}\right) \left(\frac{z_{\alpha} + z_{\beta}}{z_1 - z_2}\right)^2. \quad (5)$$

In relations (4) and (5) the values of  $z_{\alpha}$ ,  $z_{\beta}$ ,  $z_1$  and  $z_2$  represent the  $(1 - \alpha)*100$ ,  $(1 - \beta)*100$ ,  $(1 - p_1)*100$  and  $(1 - p_2)*100$  percentiles of the standard normal distribution respectively.

By  $\alpha$  and  $\beta$  there are denoted the producer's risk value and the customer's risk value and by  $p_1$  and  $p_2$  there are denoted the Acceptable Quality Level *AQL* and the Rejectable Quality Level *RQL*.

## 2. Objectives

The design and application of the single sampling plans by variables in the case when the lot standard deviation  $\sigma$  is unknown require difficult calculations. Another problem is that there exist several cases for the plan application each of them having its own procedure that must be followed.

The objective of the paper is to develop an original computer program that can be used for design and application of this type of plans. The program selects the appropriate case of plan application, calculates the plan parameters and by means of the measurements recorded by the user, it will make the decision about lot acceptance or rejection.

## 3. Methods

The computer program that will be used for the plan design and application was developed in Visual Basic for Applications for Excel 2019.

The program procedures were created according to the syntaxes presented in [1].

At the program run, first the *Create Plan* dialog box (Figure 1) is displayed on the screen.

The user of the program must record the acceptable quality level  $p_1$ , the rejectable quality level  $p_2$ , the producer's risk  $\alpha$  and the customer's risk  $\beta$ .

Besides these values, the user must record at least one specification limit: the upper specification limit  $U$ , or the lower specification limit  $L$ , or both limits (in the presented example the quality characteristic has an upper and a lower specification limits).

Then, the user must click the command button *Calculate Plan*. This action will run the procedure *Calculate\_Plan\_Click()* that uses the formula (4) for calculating the critical distance  $k$  and formula (5) for calculating the sample size  $n$ . The maximum standard deviation is calculated according to formula (3). These results will be recorded into the

worksheet labelled *Plan*. The procedure will also create a table that will be used by the user for recording the sample measurements. The table will have  $n$  rows.

Fig. 1. The *Create Plan* dialog box

The listing for the part of procedure *Calculate\_Plan\_Click()* that corresponds to the calculation of the plan parameters  $n$  and  $k$  is presented in Figure 2:

```

p1 = Val(tbx_p1.Text)
p2 = Val(tbx_p2.Text)
alpha = Val(tbx_alpha.Text)
beta = Val(tbx_beta.Text)
If Len(tbx_U.Text) > 0 Then
    U = Val(tbx_U.Text)
Else
    U = " -"
End If
If Len(tbx_L.Text) > 0 Then
    L = Val(tbx_L.Text)
Else
    L = " -"
End If
z_1 = wf.Norm_S_Inv(1 - p1)
z_2 = wf.Norm_S_Inv(1 - p2)
z_alpha = wf.Norm_S_Inv(1 - alpha)
z_beta = wf.Norm_S_Inv(1 - beta)
k = (z_alpha * z_2 + z_beta * z_1) / (z_alpha + z_beta)
n = wf.RoundUp((1 + k * k / 2) * ((z_alpha + z_beta) / (z_1 - z_2)) ^ 2, 0)

```

Fig. 2. The calculation of the plan parameters

The listing for the part of procedure *Calculate\_Plan\_Click()* that corresponds to the calculation of *MSD* is presented in Figure 3:

```

If Not (Len(tbx_L.Text) = 0 Or Len(tbx_U.Text) = 0) Then
    zp_star = k
    p_2star = 1 - wf.Norm_S_Dist(zp_star, 1)
    zp_2star = wf.Norm_S_Inv(1 - p_2star / 2)
    MSD = (U - L) / (2 * zp_2star)
    Range("A10").Value = "zp* ="
    Range("B10").Value = zp_star
    Range("A11").Value = "p** ="
    Range("B11").Value = p_2star
    Range("A12").Value = "zp** ="
    Range("B12").Value = zp_2star
    Range("A13").Value = "MSD ="
    Range("B13").Value = MSD
    Range("A10:B13").Borders.LineStyle = xlContinuous
End If

```

Fig. 3. The calculation of *MSD*

The result of the run of procedure is displayed in Figure 4. Everything except the sample measurements was completed on the worksheet by the procedure.

	A	B	C	D	E	F	G	H	I
1	Single sampling plan by variables with unknown sigma								
2			L =	80					Decision
3			U =	80.05					
4									
5									
6	Plan parameters:				Please record the measurements:				
7	k =	1.759463			#	x			
8	n =	41			1	80.034			
9					2	80.017			
10	zp* =	1.759463			3	80.030			
11	p** =	0.039249			4	80.012			
12	zp** =	2.061562			5	80.020			
13	MSD =	0.012127			6	80.024			
					.....				
46					39	80.010			
47					40	80.026			
48					41	80.026			

Fig. 4. The plan parameters and the sample

In Figure 4 there are presented only the first and the last part of the table used for recording the sample measurements. The user of program must record the sample measurements, and then has to click the command button labelled *Decision*. This action will run a procedure that will make the decision about the lot acceptance or rejection. Depending on the specification limits recorded by the user, this procedure will call one of the following three procedures: a procedure for a plan with an upper specification

limit, or a procedure for a plan with a lower specification limit, or a procedure for a plan with two specification limits. In Figure 5, it is presented the most important part of the listing for the procedure that makes the decision for the plan with two specification limits. The other two procedures were created analogously.

```

Range("I7").Value = "xbar ="
xbar = wf.Average(Range("F:F"))
Range("J7").Value = xbar
Range("I8").Value = "s ="
s = wf.StDev_S(Range("F:F"))
Range("J8").Value = s
Range("I7:J8").Borders.LineStyle = xlContinuous
MSD = Range("B13").Value
If s > MSD Then
    MsgBox "The lot is rejected (s > MSD)", vbInformation, "Decision"
    Exit Sub
End If
Range("I10").Value = "Two plans with one single specification will be applied"
L = Range("D2").Value
U = Range("D3").Value
k = Range("B7").Value
ZL = (xbar - L) / s
ZU = (U - xbar) / s
Range("I11").Value = "ZL ="
Range("J11").Value = ZL
Range("I12").Value = "ZU ="
Range("J12").Value = ZU
If ZL >= k And ZU >= k Then
    Range("I13").Value = "The lot is accepted"
Else
    Range("I13").Value = "The lot is rejected"
End If
Range("I11:J13").Borders.LineStyle = xlContinuous

```

Fig. 5. *The procedure for the plan with two specification limits*

In the presented case, the decision was made by the procedure for a plan with two specification limits. This procedure calculated first the sample mean and the sample standard deviation. Then, because the condition  $s \leq MSD$  was fulfilled, two plans with one single specification limit were applied. There were calculated the statistics  $z_L$  and  $z_U$  and were compared to the critical distance  $k$ . In this case, the lot was accepted (Figure 6):

xbar =	80.02327			
s =	0.009046			
Two plans with one single specification will be applied				
ZL =	2.572689			
ZU =	2.954464			
The lot is accepted				

Fig. 6. *The decision*

#### 4. Results and Discussions

The developed program was presented for an operating case when the quality characteristic had two specification limits. The value of the sample standard deviation  $s$  was less than  $MSD$ , so the lot was accepted. If  $s$  were greater than  $MSD$ , the lot were rejected by the program. In Figure 7 it is presented a case when the sample standard deviation  $s$  were greater than  $MSD$  (the figure presents only a part of the sample). The inspection conditions were:  $p_1 = 0.012$ ,  $p_2 = 0.055$ ,  $\alpha = 0.05$ ,  $\beta = 0.1$ ,  $U = 50.05$  and  $L = 50$ .

	A	B	C	D	E	F	G	H	I	J
1	Single sampling plan by variables with unknown sigma									
2			L =	50						Decision
3			U =	50.05						
4										
5										
6	Plan parameters:				Please record the measurements:					
7	k =	1.886759			#	x			xbar =	50.02545
8	n =	55			1	50.020			s =	0.012648
9					2	50.030				
10	z <sub>p</sub> * =	1.886759			3	50.033				
11	p** =	0.029596			4	50.051				
12	z <sub>p</sub> ** =	2.17545			5	50.025				
13	MSD =	0.011492			6	50.024				
14					7	50.024				
15					8	50.030				
16					9	50.016				

Decision

**i** The lot is rejected (s > MSD)

OK

Fig. 7. The lot is directly rejected because the sample standard deviation  $s$  is too high

The presented program may also be used for design and application of acceptance sampling plans for quality characteristics with one single specification limit.

Next, the paper presents a case when the quality characteristic has only an upper specification limit. The case when the quality characteristic has a lower specification limit is analogous. The inspection conditions are: the acceptable quality level  $p_1 = 0.015$ , the rejectable quality level  $p_2 = 0.045$ , the producer's risk  $\alpha = 0.05$  and the customer's risk  $\beta = 0.1$ . The upper specification limit is  $U = 25$ . The results of the plan design and application are presented in Figure 8:

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Single sampling plan by variables with unknown sigma												
2			L =	-									Decision
3			U =	25									
4													
5													
6	Plan parameters:				Please record the measurements:								
7	k =	1.903278			#	x			xbar =	24.98047			
8	n =	107			1	24.966			s =	0.015166			
9					2	24.970							
10					3	24.995							
11					4	24.967							
12					5	24.982							
13					6	24.971							

Plan with one upper specification U will be applied

ZU = 1.287625

The lot is rejected

Fig. 8. A plan with an upper specification limit

## 5. Conclusions

The computer program presented by this paper is original and can be applied for calculating the plan parameters and at the application of single sampling plans by variables when the lot standard deviation  $\sigma$  is unknown.

The advantages of the developed program are the following:

- The program performs all the calculations and also makes the decision about the lot acceptance or rejection;
- The interface is user friendly and the input data are very easy to record;
- The user of the program must not have knowledge about Statistics (if the plan design and application are performed without a computer program this knowledge is required);
- The program can be applied for quality characteristics with one or two specification limits.

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